

Seat No. : \_\_\_\_\_

**DD-115**

**December-2018**

**B.C.A., Sem.-I**

**CC-104 : Fundamental Mathematical Concepts (FMC)**

**Time : 2:30 Hours]**

**[Max. Marks : 70**

1. (A) (1) Let  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{2, 3, 4, 5, 6\}$  and  $C = \{1, 2, 4, 5, 6, 8\}$ . 7
- (a) Find :  $B \cup C$ .
  - (b) If  $f : A \rightarrow B$  is an onto function, then find the range of  $f$ .
  - (c) Write the domain and c-domain of the function  $g : (A - B) \rightarrow (B - A)$
  - (d) Find :  $(A - C) \cap B$ .
  - (e) Find :  $(A - B) \Delta (B - A)$ .
  - (f) Find :  $(A \cap B) \times (B \cap A)$ .
  - (g) Verify that  $A - (B \cup C) = (A - B) \cap (A - C)$
- (2) Write any three properties of logarithmic function. Also show that, if  $\log \left( \frac{x-y}{3} \right) = \frac{1}{2} (\log x + \log y)$  then  $x^2 + y^2 = 11xy$ , ( $x > y > 0$ ). 7

**OR**

- (1) Let  $U = \{x \in \mathbb{Z} \mid 1 \leq x^2 \leq 10\}$ ,  $A = \{x \in U \mid -2 < x \leq 3\}$ ,  $B = \{x \in U \mid 1 \leq x^3 < 10\}$ . For these sets answer the following questions.
- (a) Find,  $(A \cup B)'$
  - (b) Find,  $U - (A \cup B)$
  - (c) Find,  $(A - B) \times (B - A)$
  - (d) Find,  $n(A \times B)$
- (2) (a) Find the Domain and Range of the function  $f : \mathbb{N} \rightarrow \mathbb{N}$ ,  $f(x) = x + 1$ . Is this function one to one ?
- (b) If  $f(x) = \log \left( \frac{1+x}{1-x} \right)$ , then prove that  $f \left( \frac{2x}{1+x^2} \right) = 2f(x)$ .

(B) Do as Directed : (Any **Four**)

**4**

- (1) Let  $A$  be an empty set. How many elements in a set  $P(A)$  ?
- (a) 0
  - (b) 1
  - (c) 2
  - (d) None of these
- (2) The elements of a set  $A = \{x \mid x^2 - 3x + 2 = 0, x \in \mathbb{Z}\}$  are
- (a)  $\{1, 2, 3\}$
  - (b)  $\{-1, 1\}$
  - (c)  $\{1, 2\}$
  - (d) None of these

- (3) If  $f: \mathbb{N} \rightarrow \mathbb{N}$ ,  $f(x) = x + 1$ , then the domain and co-domain are  
 (a)  $\mathbb{N}, \mathbb{N}$  (b)  $\mathbb{N} \cup \{0\}, \mathbb{N}$  (c)  $\mathbb{N}, \mathbb{N} \cup \{0\}$  (d) None of these
- (4) If  $f: \mathbb{N} \rightarrow \mathbb{Z}$ ,  $f(x) = x - 2$ , then  $R_f$  = the range of the function  $f$  is,  
 (a)  $\mathbb{N}$  (b)  $\mathbb{N} \cup \{0\}$  (c)  $\mathbb{N} - \{1\}$  (d)  $\mathbb{Z} - \{0\}$
- (5) A function If  $f: \mathbb{N} \rightarrow \mathbb{N}$ ,  $f(x) = x$  is one-one and onto function. (True / False)
- (6) A relation If  $f: \mathbb{N} \rightarrow \mathbb{N}$ ,  $f(x) = x - 1$  is a function. (True / False)

2. (A) (1) Let  $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 2 & -1 \\ 3 & -2 & 1 \end{bmatrix}$  then verify that

$$(A + B)^2 = A^2 + 2AB + B^2.$$

7

- (2) Solve the following system using Matrix inversion method.

7

$$x + 2y + 3z = 6$$

$$2x + 4y + z = 7$$

$$3x + 2y + 9z = 14$$

**OR**

- (1) Define rank of a matrix. Also find the rank of following matrices :

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 2 & -4 \\ 1 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- (2) If  $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -2 \\ 4 & -2 & 3 \end{bmatrix}$ , then find symmetric and skew-symmetric matrices using  $A$ .

- (B) Do as Directed (Any **Four**) :

4

- (1) Let  $A = [a_{ij}]$  be a Square Matrix. Then it is an identity matrix if  
 (a)  $a_{ij} = 0$  for all  $i$  and  $j$  (b)  $a_{ij} = 1$  for all  $i$  and  $j$   
 (c)  $a_{ij} = 0$  for all  $i \neq j$  (d) None of these
- (2) Rank of a Null Matrix of order 3 is  
 (a) 1 (b) 2 (c) 3 (d) 0
- (3) A matrix  $A$  is invertible if  
 (a)  $|A| = 0$  (b)  $|A| \neq 0$   
 (c)  $\text{adj } A = \text{Null matrix}$  (d) None of these
- (4) If the determinant of a matrix  $A$  is 2 then the determinant of  $A^T$  is equals to  
 (a) 2 (b) -2 (c) 0 (d) None of these
- (5) All diagonal entries of a Skew-symmetric matrix is zero. (True /False)
- (6) Inverse of an identity matrix is itself. (True/False)

3. (A) (1) Let four points A(3, 2), B(5, 4), C(3, 6) and D(1, 4) on XY-plane. Find the equation of lines  $\overleftrightarrow{AB}$ ,  $\overleftrightarrow{BC}$ ,  $\overleftrightarrow{CD}$  and  $\overleftrightarrow{AD}$ . Explain the diagram made by these lines mathematically. 7
- (2) Find the equation of a line which passes through the point (4, 10) and makes an angle of  $45^\circ$  with the line  $x - 3y + 8 = 0$ . 7

**OR**

- (1) If (4, 3), (6, 5) and (2, 4) are the mid points of sides of a triangle, find the vertices of the triangle.
- (2) Find the area of a triangle formed by lines given below :  
 $7x + y - 11 = 0$ ,  $x + 3y + 7 = 0$ ,  $3x - y + 1 = 0$ .
- (B) Do as Directed : (Any **Three**) 3
- (1) Two line  $y = 3$  and  $x = 3$  are,  
 (a) Parallel (b) Perpendicular  
 (c) Making angle of  $45^\circ$  (d) None of these
- (2) An equation of a line passing through (0, 0) and making angle  $45^\circ$  with x-axis is  
 (a)  $x - y = 0$  (b)  $x + y = 0$  (c)  $x - y = 1$  (d)  $x + y = 1$
- (3) Area of a diagram made by joining three points (1, 1), (2, 2) and (3, 3) is,  
 (a) 0 (b) 1 (c) 4 (d) 9
- (4) Equation of a line passing through (0, 0) and having slope  $m = -1$  is,  
 (a)  $x - y = 0$  (b)  $x + y = 0$  (c)  $x - y = 1$  (d)  $x + y = 1$
- (5) Two lines are parallel if  $m_1 = m_2$ . (True / False)

4. (A) (1) (i) Evaluate following limit :  $\lim_{x \rightarrow 1} \frac{x^7 - 1}{x^4 - 1}$ .

(ii) Evaluate :  $\int_{-1}^1 (x^2 + 2x + 1) dx$  7

- (2) Find  $\frac{dy}{dx}$  when  $y = \log(2x + 3)^5$ . 7

**OR**

- (1) (i) Find  $\frac{dy}{dx}$  for given  $y = 1 + x + x^2 + x^3 + x^4$
- (ii) Evaluate :  $\int (3x + 4)^5 dx$
- (2) Check the continuity of given function  $f(x)$  given below at  $x = 3$ .

$$f(x) = \frac{x^2}{5}, \quad x < 5$$

$$= x^2 - 20, \quad x \geq 3$$

(B) Do as Directed : (Any **Three**)

**3**

- (1) The derivative of the constant function is,  
(a) 0 (b) 1 (c) 2 (d) None of these
- (2) As  $x \rightarrow \infty$ , the constant function  $f(x) = 3$ , tends to,  
(a) 3 (b) 0 (c)  $\infty$  (d) None of these
- (3) Integration of a function  $f(x) = 0$  is,  
(a) 0 (b) 1 (c) 2 (d) None of these
- (4) Area bounded by the curve  $f(x) = 1$ ,  $x = 0$ ,  $x = 1$  and  $y = 0$  is,  
(a) 0 (b) 1 (c) 2 (d) None of these
- (5)  $\int_0^{\log_e 2} e^x dx = \underline{\hspace{2cm}}$ .  
(a) 0 (b) 1 (c) 2 (d) None of these

**DD-115****December-2018****B.C.A., Sem.-I****CC-104 : Basics of Mathematics (BM)****Time : 2:30 Hours]****[Max. Marks : 70**

1. (A) (1) (i) For three sets  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{2, 4, 6, 8\}$  and  $C = \{1, 3, 4, 5, 8\}$ , verify that  $A - (B \cap C) = (A - B) \cup (A - C)$ . 7

(ii) If  $f(x) = \log \left( \frac{1+x}{1-x} \right)$ , then prove that  $f \left( \frac{2x}{1+x^2} \right) = 2f(x)$ .

- (2) (i) If  $A = \{x / |x^3 - 2| \leq 25, x \in \mathbb{N}\}$ ,  $B = \{y / 1 < y < 5, x \in \mathbb{N}\}$  and  $C = \{z/z^4 = 81, x \in \mathbb{N}\}$  then verify that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ . 7

- (ii) Define following terms :

Range of a function, onto function

**OR**

- (1) (i) If  $A = \{5, 7\}$ ,  $B = \{7, 8\}$ ,  $C = \{5, 8\}$  then verify that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$ .
- (ii) If the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2 + 1$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $g(x) = \frac{x}{x+1}$ . Find fog and gof if they exist.
- (2) (i) Let  $A = \{x / x \in \mathbb{N}\}$ ,  $B = \{x / x = 2n, n \in \mathbb{N}\}$ ,  $C = \{x / x = 2n - 1, n \in \mathbb{N}\}$  and  $D = \{x / x \text{ is a prime number}\}$ . Find (i)  $A \cup B$  (ii)  $C \cap D$  (iii)  $D - A$
- (ii) If  $\log \left( \frac{x+y}{3} \right) = \frac{1}{2} (\log x + \log y)$  then verify that  $x^2 + y^2 = 11xy$ ,  $(x > y > 0)$ .

- (B) Do as Directed : (Any **Four**)

**4**

- (1) If A is a singleton set, then its power set is empty. (True / False)
- (2) Every set is a subset of itself. (True / False)
- (3) A constant function has only one element in its range. (True / False)
- (4) If  $f : \mathbb{N} \rightarrow \mathbb{Z}$ ,  $f(x) = x - 2$ , then  $R_f$  = the range of the function f is,  
 (a)  $\mathbb{N}$  (b)  $\mathbb{N} \cup \{0\}$  (c)  $\mathbb{N} - \{1\}$  (d)  $\mathbb{Z} - \{0\}$
- (5) A function If  $f : \mathbb{N} \rightarrow \mathbb{N}$ ,  $f(x) = x$  is one-one and onto function. (True / False)
- (6) A relation If  $f : \mathbb{N} \rightarrow \mathbb{N}$ ,  $f(x) = x - 1$  is a function. (True / False)

2. (A) (1) (i) Solve the following system using Cramer's Rule. 7

$$2x - 2y + z = 1$$

$$x + 2y + 2z = 2$$

$$2x + y - 2z = 7$$

(ii) If  $A = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 5 & 6 \\ 1 & 1 & 2 \end{bmatrix}$  then verify that  $AA^{-1} = I$ .

- (2) (i) Solve the following system using inversion Method : 7

$$x + y + z = 3$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 6$$

(ii) Find the rank of matrices,  $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 2 & 4 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 2 & 2 \\ 1 & 4 & 3 \end{bmatrix}$ .

**OR**

- (1) (i) Let A be a square matrix then prove that A can be written as a sum of symmetric and skew-symmetric matrices.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

(ii) If  $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ , verify that  $A^2 - 7A - 2I = 0$ , where I is of order 2.

- (2) (i) For the following two matrices A and B verify that  $(A + B)^T = A^T + B^T$ .

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$$

(ii) If  $A = \begin{bmatrix} 6 & -5 \\ -3 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 17 \\ 1 \end{bmatrix}$ , then find a matrix X such that  $AX = B$ .

- (B) Do as Directed : (Any **Four**) 4

(1) If  $A = [a_{ij}]$  is a Square Matrix, then its inverse always exists. (True / False)

(2) Rank of a Null Matrix of order 3 is

(a) 1                      (b) 2                      (c) 3                      (d) 0

(3) Inverse of an identity matrix is itself. (True / False)

- (4) If the determinant of a matrix  $A$  is 0 then the determinant of  $A^T$  is equals to  
 (a) 2 (b) -2 (c) 0 (d) None of these
- (5) Every diagonal matrix is a square matrix. (True /False)
- (6) If a matrix  $A = [a_{ij}]$  with  $a_{ij} = 1$  for all  $i$  and  $j$ , then the inverse of  $A$  does not exist. (True / False)

3. (A) (1) (i) Find the area of triangle having vertices at (4, 4), (3, -2), (3, -16). 7  
 (ii) Show that the points (6, 6), (2, 3) and (4, 7) are the vertices of a right angled triangle.
- (2) (i) Show that the points (-1, 1), (3, -2) and (-5, 4) are collinear. 7  
 (ii) If a point  $P(1, 2)$  divides a line segment joining points  $A(-2, -1)$  and  $B$  in the ratio 2 : 3 then find the coordinates of point  $B$ .

**OR**

- (1) (i) Find the equation of a line passing through the intersection of lines  $l_1 : x - 1 = 0$  and  $l_2 : y - 1 = 0$  and parallel to  
 (ii) In which ratio a point  $P(0, 0)$  divides a line segment joining two points (2, 2) and (3, 3).
- (2) (i) Find the equation of a line which cuts off equal intercepts on the coordinate axes and passing through the point  $P(1, 2)$ .  
 (ii) Determine  $x$  so that 5 is the slope of the line through  $(x, 12)$  and  $(3, 2)$ .

(B) Do as Directed : (Any **Three**) 3

- (1) Two line  $y = 4$  and  $y = 7$  are,  
 (a) Parallel (b) Perpendicular  
 (c) Making angle of  $45^\circ$  (d) None of these
- (2) An equation of a line passing through (0, 0) and (1, 1) is having slope positive. (True / False)
- (3) Two lines are parallel if their slopes are same. (True / False)
- (4) Equation of a line passing through (0, 0) and having slope  $m = -1$  is,  
 (a)  $x - y = 0$  (b)  $x + y - 0$  (c)  $x - y = 1$  (d)  $x + y = 1$
- (5) Intersection point of  $x - y = 0$  and  $x + y = 0$  is  
 (a) (0,0) (b) (1,1) (c) (0,1) (d) (1,0)

4. (A) (1) (i) Evaluate following limit : 7

$$\lim_{x \rightarrow 0} \frac{\sqrt{2+3x} - \sqrt{2-5x}}{4x}$$

(ii) Find  $\frac{dy}{dx}$  for given  $y = \left(x + \frac{1}{x}\right)^3$

- (2) (i) Check the continuity of given function  $f(x)$  given below at  $x = 3$ . 7

$$f(x) = \frac{x^2 - 9}{x - 3}, \quad x < 3$$

$$= 2x - 3, \quad x \geq 3$$

(ii) Find  $\frac{dy}{dx}$  when  $y = e^{2x+3}$ .

**OR**

(1) (i)  $\int \sec^2 x \tan x \, dx$

(ii) Evaluate :  $\int_{-1}^1 (3x^2 + 2^x) \, dx$

(2) (i) Evaluate :  $\int \frac{1}{4x+5} \, dx$

(ii)  $\int_1^4 \left(x + \frac{1}{x}\right) \, dx$

- (B) Do as Directed : (Any **Three**) 3

- (1) The derivative of  $y = 5$  with respect to  $x$  is,  
 (a) 0 (b) 1 (c) 2 (d) None of these
- (2) Derivative of a constant function is,  
 (a) 0 (b) 1 (c) 2 (d) None of these
- (3) Integration of a constant function  $f(x) = 1$  is,  
 (a) 0 (b) 1 (c)  $x$  (d) None of these
- (4) Area bounded by the curve  $f(x) = 1$ ,  $x = 0$ ,  $x = 1$  and  $y = 0$  is,  
 (a) 0 (b) 1 (c) 2 (d) None of these
- (5)  $\int_0^{\log_e 2} e^x \, dx = \underline{\hspace{2cm}}$ .  
 (a) 0 (b) 1 (c) 2 (d) None of these